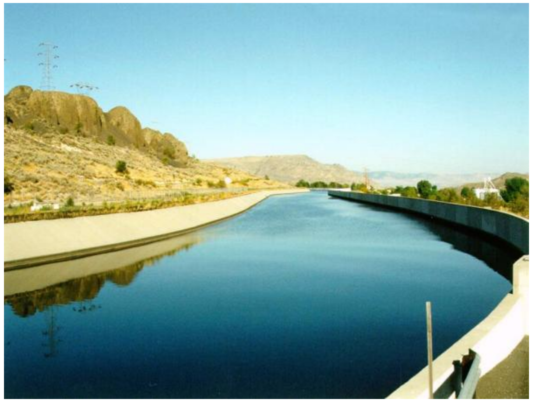
Open Channel Flow

- Liquid (water) flow with a _free surface___ (interface between water and air)



- relevant for
  - natural channels: rivers, streams
  - engineered channels: canals, sewer lines or culverts (partially full), storm drains
- of interest to hydraulic engineers
  - location of free surface
  - velocity distribution
  - discharge - stage (_depth_) relationships
  - optimal channel design

Topics in Open Channel Flow

- Uniform Flow   normal depth
  - Discharge-Depth relationships
- Channel transitions
  - Control structures (sluice gates, weirs…)
  - Rapid changes in bottom elevation or cross section
- Critical, Subcritical and Supercritical Flow
- Hydraulic Jump
- Gradually Varied Flow
  - Classification of flows
  - Surface profiles

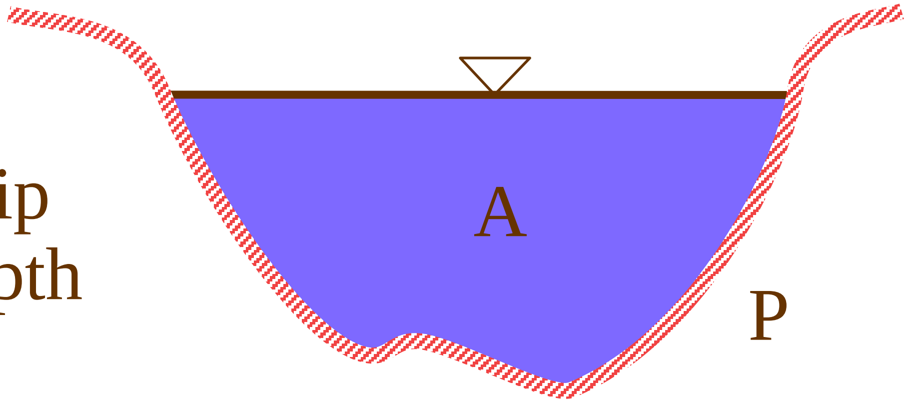
Classification of Flows

- Steady and Unsteady (Temporal)
  - Steady: velocity at a given point does not change with time
- Uniform, Gradually Varied, and Rapidly Varied (Spatial)
  - Uniform: velocity at a given time does not change within a given length of a channel
  - Gradually varied: gradual changes in velocity with distance
- Laminar and Turbulent
  - Laminar: flow appears to be as a movement of thin layers on top of each other
  - Turbulent: packets of liquid move in irregular paths

Momentum and Energy Equations

- Conservation of Energy
  - "losses" due to conversion of turbulence to heat
  - useful when energy losses are known or small
    - Contractions
  - Must account for losses if applied over long distances
    - We need an equation for losses
- Conservation of Momentum
  - "losses" due to shear at the boundaries
  - useful when energy losses are unknown
    - Expansion

Open Channel Flow: Discharge/Depth Relationship



- Given a long channel of constant slope and cross section find the relationship between discharge and depth
- Assume
  - Steady Uniform Flow - no acceleration
  - prismatic channel (no change in geometry with distance)
- Use Energy, Momentum, Empirical or Dimensional Analysis?
- What controls depth given a discharge?
- Why doesn't the flow accelerate? Force balance

$$\tau_0 = -\frac{\gamma h_l d}{4l}$$

Steady-Uniform Flow: Force Balance

Shear force = $\tau_o P \Delta x$

Wetted perimeter = P

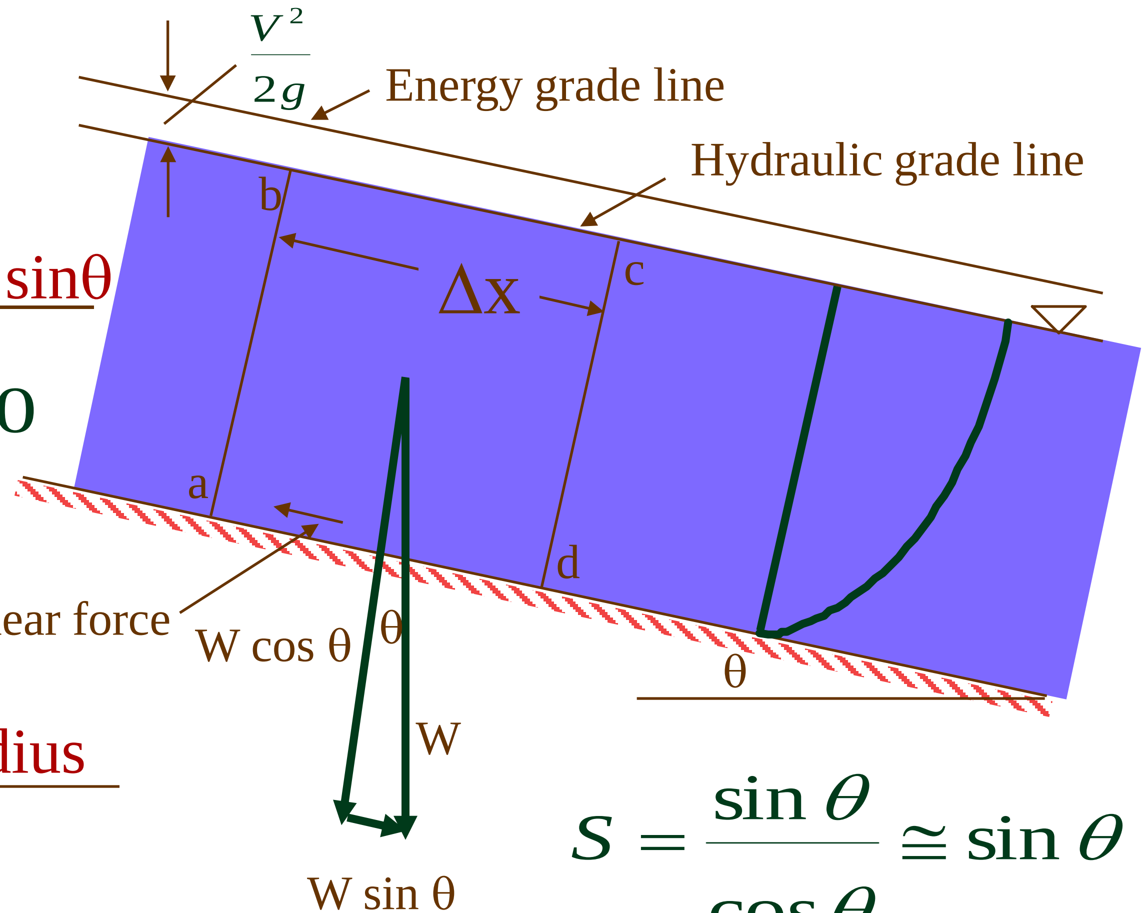
Gravitational force = $\gamma A \Delta x \sin\theta$

$$\gamma A \Delta x \sin\theta - \tau_o P \Delta x = 0$$

$$\tau_o = \gamma \frac{A}{P} \sin\theta$$

$$\frac{A}{P} = R_h$$ Hydraulic radius

$$t_o = g R_h S$$



$$S = \frac{\sin\theta}{\cos\theta} \cong \sin\theta$$

Relationship between shear and velocity? Turbulence

Open Conduits: Dimensional Analysis

- Geometric parameters
  - Hydraulic radius (R_h)
  - Channel length (l)
  - Roughness (ε)

$$R_h = \frac{A}{P}$$

- Write the functional relationship

$$C_p = f\left(\frac{l}{R_h}, \frac{\varepsilon}{R_h}, \text{Re}, \cancel{Fr}, \cancel{M}, \cancel{W}\right)$$

- Does Fr affect shear? No!

$$Fr = \frac{V}{\sqrt{yg}}$$

Pressure Coefficient for Open Channel Flow?

$$C_p = \frac{-2\Delta p}{\rho V^2}$$

Pressure Coefficient

(Energy Loss Coefficient)

$$-\Delta p = \gamma h_l$$

$$C_{h_l} = \frac{2gh_l}{V^2}$$

Head loss coefficient

$$h_l = S_f l$$

Friction slope

$$C_{S_f} = \frac{2gS_f l}{V^2}$$

Friction slope coefficient

Slope of EGL

Dimensional Analysis

$$C_{S_f} = f\left(\frac{l}{R_h}, \frac{e}{R_h}, \mathrm{Re}\right)$$

$$C_{S_f} = \frac{2gS_f l}{V^2}$$

$$C_{S_f} = \frac{l}{R_h} f\left(\frac{e}{R_h}, \mathrm{Re}\right)$$

Head loss ∝ length of channel

$$C_{S_f}\frac{R_h}{l} = f\left(\frac{e}{R_h}, \mathrm{Re}\right) = l$$

(like f in Darcy-Weisbach)

$$C_{S_f}\frac{R_h}{l} = l$$

$$h_l = \mathrm{f}\,\frac{L}{D}\frac{V^2}{2g}$$

$$\frac{2gS_f l}{V^2}\frac{R_h}{l} = l$$

$$S_f = \frac{l}{R_h}\frac{V^2}{2g}$$

$$V = \sqrt{\frac{2gS_f R_h}{l}}$$

$$V = \sqrt{\frac{2g}{l}}\sqrt{S_f R_h}$$

Chezy Equation (1768)

- Introduced by the French engineer Antoine Chezy in 1768 while designing a canal for the water-supply system of Paris

$$V = C\sqrt{R_h S_f} \qquad \text{compare} \qquad V = \sqrt{\frac{2g}{f}}\sqrt{S_f R_h}$$

where C = Chezy coefficient

$$60\,\frac{\sqrt{m}}{s} < C < 150\,\frac{\sqrt{m}}{s}$$

$$0.0054 > f > 0.00087$$

$$0.022 > f > 0.0035$$

For a pipe

$$d = 4R_h$$

where 60 is for rough and 150 is for smooth

also a function of **R** (like f in Darcy-Weisbach)

Darcy-Weisbach Equation (1840)

f = Darcy-Weisbach friction factor

$$R_h = \frac{A}{P} = \frac{\left(\frac{\pi d^2}{4}\right)}{\pi d} = \frac{d}{4}$$

$$h_l = f\,\frac{l}{d}\,\frac{V^2}{2g} \longrightarrow h_l = f\,\frac{l}{4R_h}\,\frac{V^2}{2g}$$

$$S_f\,l = f\,\frac{l}{4R_h}\,\frac{V^2}{2g} \longrightarrow S_f R_h = f\,\frac{V^2}{8g} \longrightarrow V = \sqrt{\frac{8g}{f}}\sqrt{S_f R_h}$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{12R_h} + \frac{2.5}{Re\sqrt{f}}\right)$$

Similar to Colebrook

For rock-bedded streams

where d_{84} = rock size larger than 84% of the rocks in a random sample

$$f = \frac{1}{\left(1.2 + 2.03\log\left[\frac{R_h}{d_{84}}\right]\right)^2}$$

Manning Equation (1891)

- Most popular in U.S. for open channels

$$V = \frac{1}{n} R_h^{2/3} S_o^{1/2}$$

(MKS units!)

Dimensions of *n*? $T/L^{1/3}$

Is *n* only a function of roughness? NO!

$$V = \frac{1.49}{n} R_h^{2/3} S_o^{1/2}$$

(English system)

Bottom slope

$$Q = VA$$

$$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2}$$

very sensitive to *n*

Values of Manning *n*

Lined Canals	n
Cement plaster	0.011
Untreated gunite	0.016
Wood, planed	0.012
Wood, unplaned	0.013
Concrete, trowled	0.012
Concrete, wood forms, unfinished	0.015
Rubble in cement	0.020
Asphalt, smooth	0.013
Asphalt, rough	0.016
Natural Channels	
Gravel beds, straight	0.025
Gravel beds plus large boulders	0.040
Earth, straight, with some grass	0.026
Earth, winding, no vegetation	0.030
Earth , winding with vegetation	0.050

n = f(surface roughness, channel irregularity, stage...)

$$n = 0.031 d^{1/6}$$ d in ft

$$n = 0.038 d^{1/6}$$ d in m

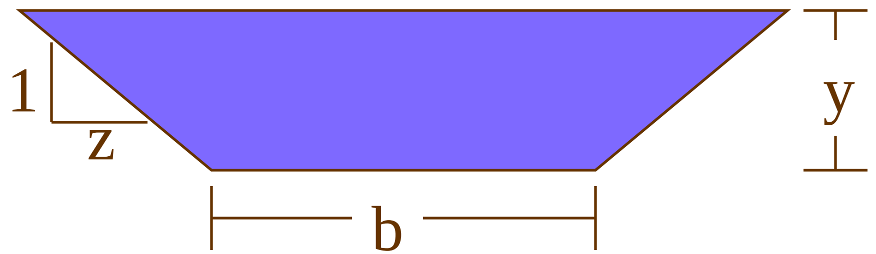
d = median size of bed material

Trapezoidal Channel

$$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2}$$

- Derive P = f(y) and A = f(y) for a trapezoidal channel
- How would you obtain y = f(Q)?

$$A = yb + y^2 z$$



$$P = 2\left[y^2 + (yz)^2\right]^{1/2} + b$$

$$P = 2y\left[1 + z^2\right]^{1/2} + b$$

Use Solver!

Flow in Round Conduits

$= (r \sin q)(r \cos q)$

$$\theta = \arccos\left(\frac{r-y}{r}\right)$$

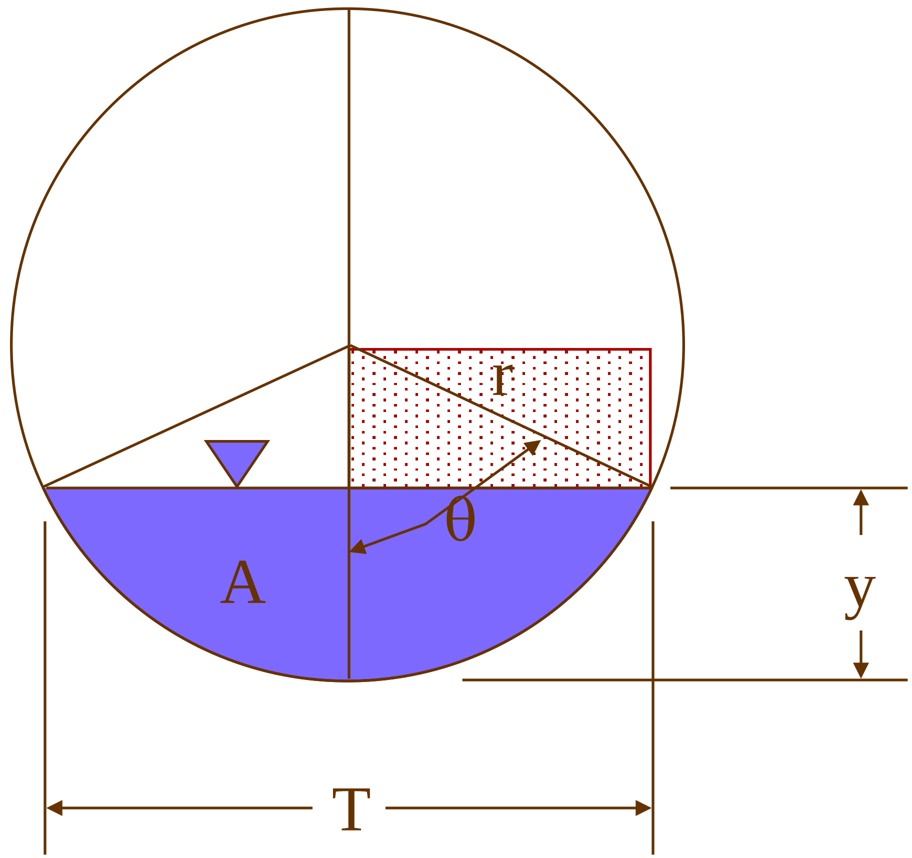
radians

$$A = r^2(\theta - \sin\theta\cos\theta)$$

$$T = 2r\sin\theta$$

$$P = 2r\theta$$

Maximum discharge when y = 0.938d



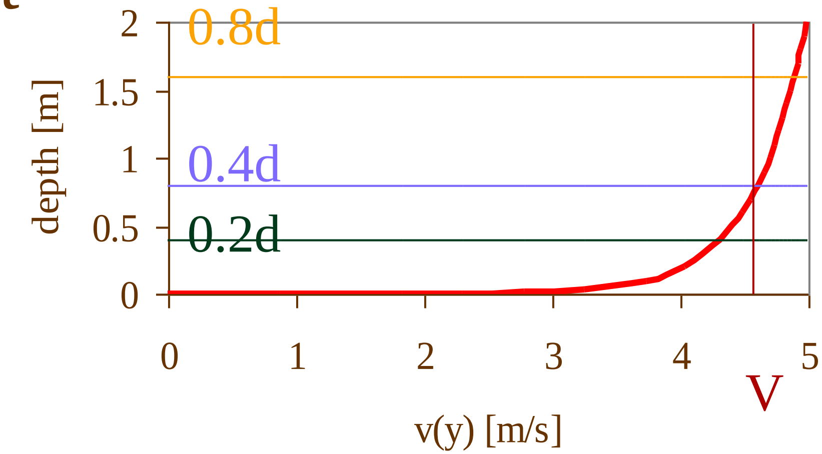
Velocity Distribution

$$v(y) = V + \frac{1}{\kappa}\sqrt{gdS_0}\left(1 + \ln\frac{y}{d}\right)$$

For channels wider than 10d

k » 0.4 Von Kármán constant

V = average velocity
d = channel depth



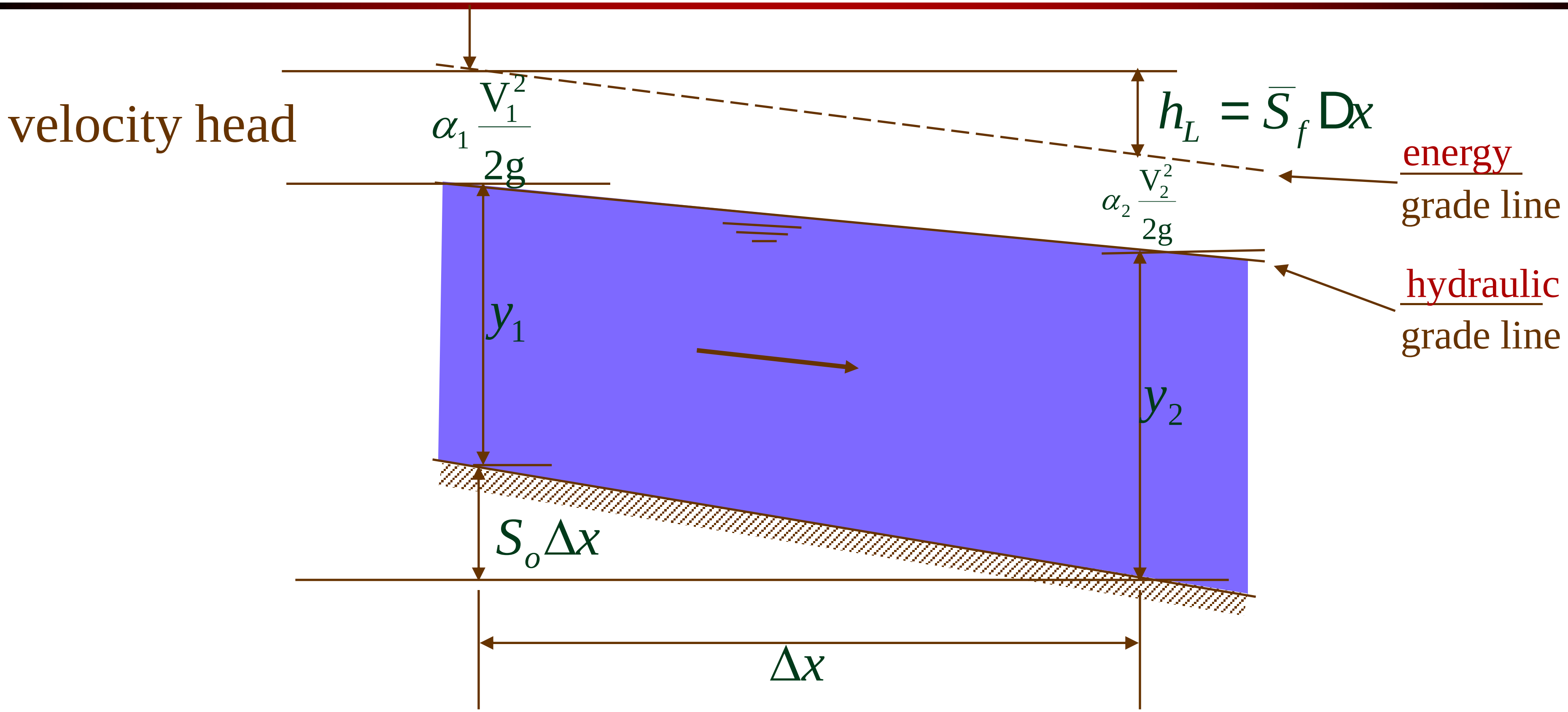
At what elevation does the velocity equal the average velocity?

$$-1 = \ln\frac{y}{d}$$

$$y = \frac{1}{e}d$$

0.368d

Open Channel Flow: Energy Relations



Bottom slope (S_o) not necessarily equal to EGL slope (S_f)

Energy Relationships

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_L$$

Pipe flow

z - measured from horizontal datum

From diagram on previous slide...

$$y_1 + S_o \Delta x + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Turbulent flow ($\alpha \cong 1$)

y - depth of flow

Energy Equation for Open Channel Flow

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Specific Energy

- The sum of the depth of flow and the velocity head is the specific energy:

+ pressure

$$E = y + \frac{V^2}{2g}$$

y - potential energy

$\frac{V^2}{2g}$ - kinetic energy

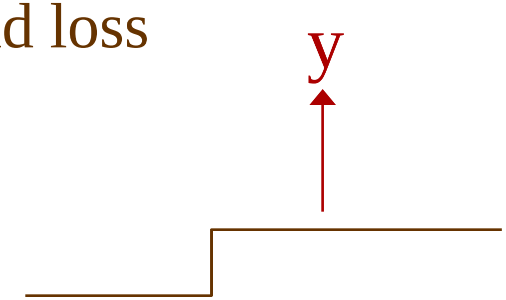
$$E_1 + S_o \Delta x = E_2 + S_f \Delta x$$

If channel bottom is horizontal and no head loss

$$E_1 = E_2$$

For a change in bottom elevation

$$E_1 - Dy = E_2$$



Specific Energy

In a channel with constant discharge, Q

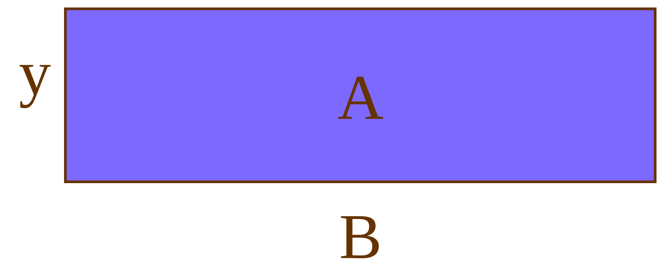
$$Q = A_1 V_1 = A_2 V_2$$

$$E = y + \frac{V^2}{2g} \longrightarrow E = y + \frac{Q^2}{2gA^2}$$ where A=f(y)

Consider rectangular channel (A = By) and Q = qB

$$E = y + \frac{q^2}{2gy^2}$$

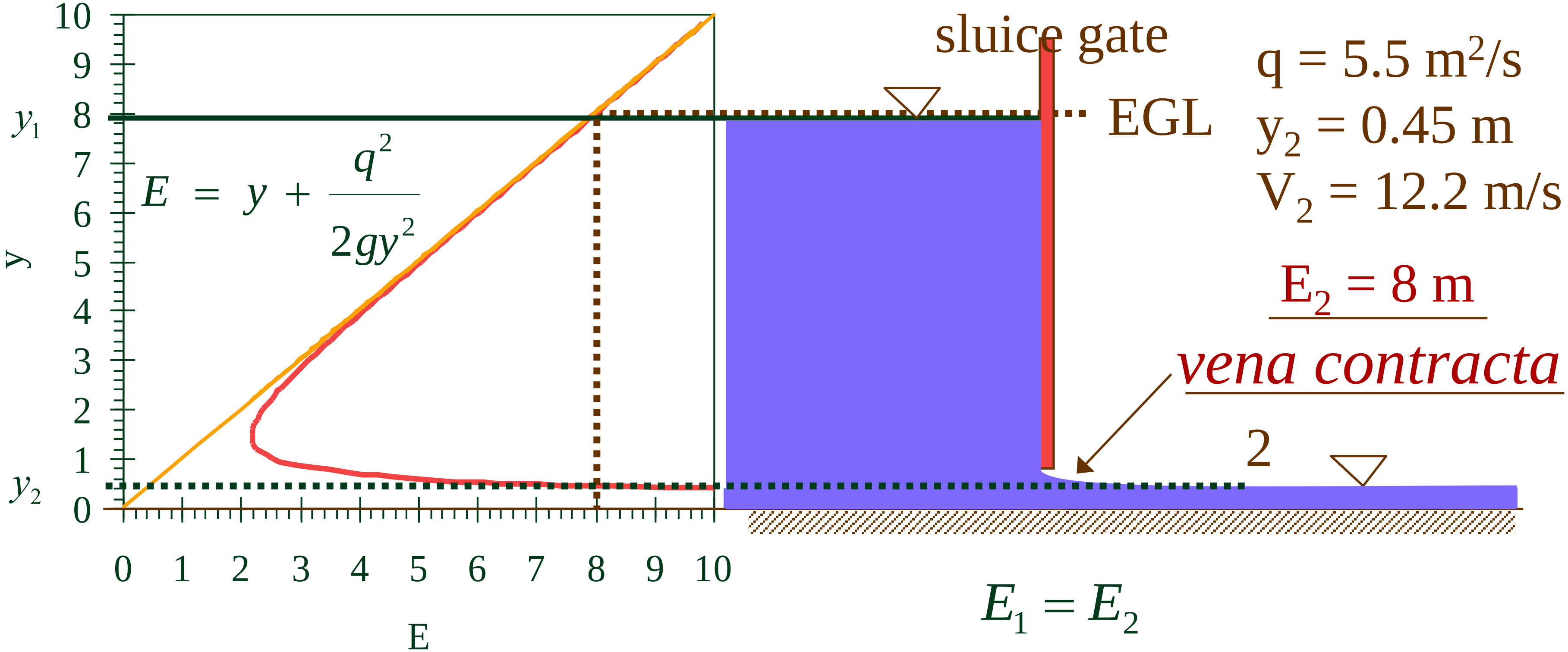
q is the discharge per unit width of channel



3 roots (one is negative)

How many possible depths given a specific energy? 2 ______

Specific Energy: Sluice Gate

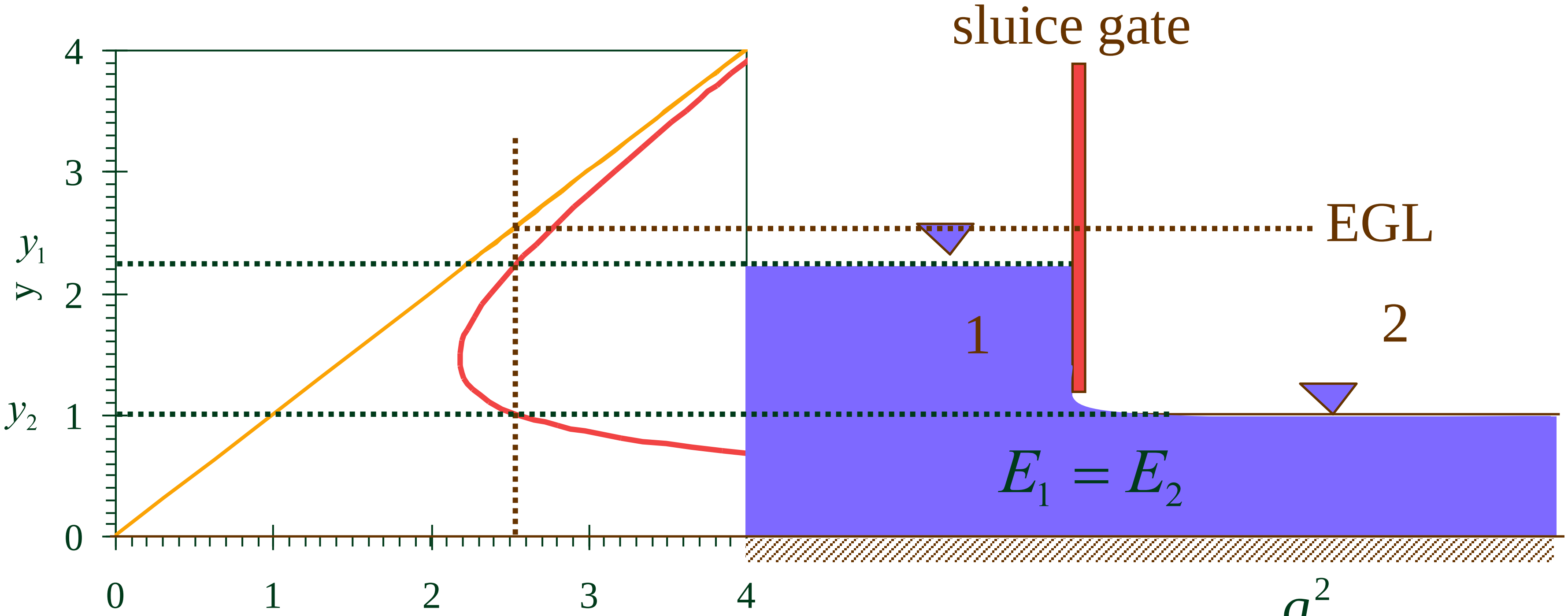


Given downstream depth and discharge, find upstream depth.

y_1 and y_2 are _alternate__ depths (same specific energy)

Why not use momentum conservation to find y_1?

Specific Energy: Raise the Sluice Gate



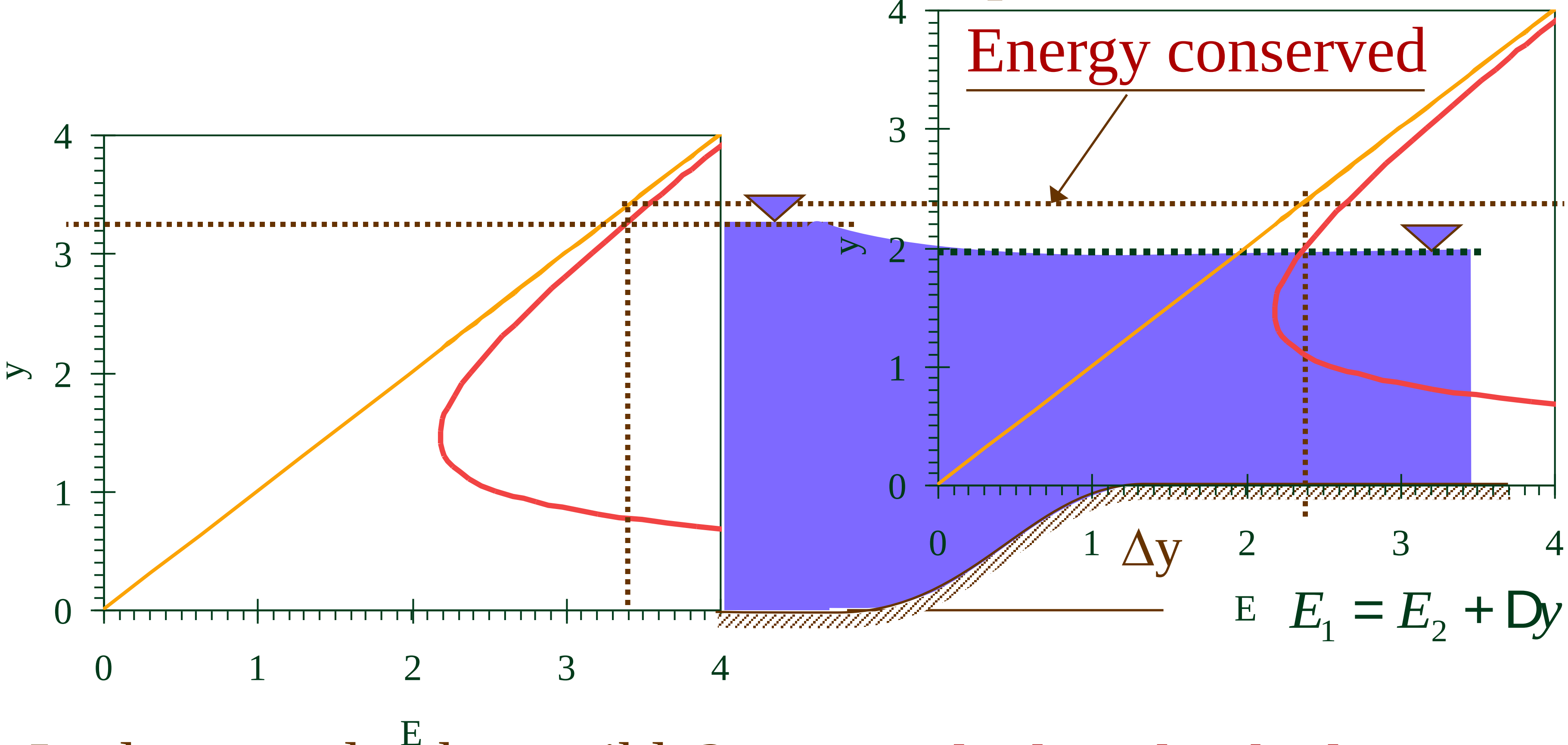
$$E = y + \frac{q^2}{2gy^2}$$

as sluice gate is raised y_1 approaches y_2 and E is minimized: Maximum discharge for given energy.

Step Up with Subcritical Flow

Short, smooth step with rise Δy in channel

Given upstream depth and discharge find y_2

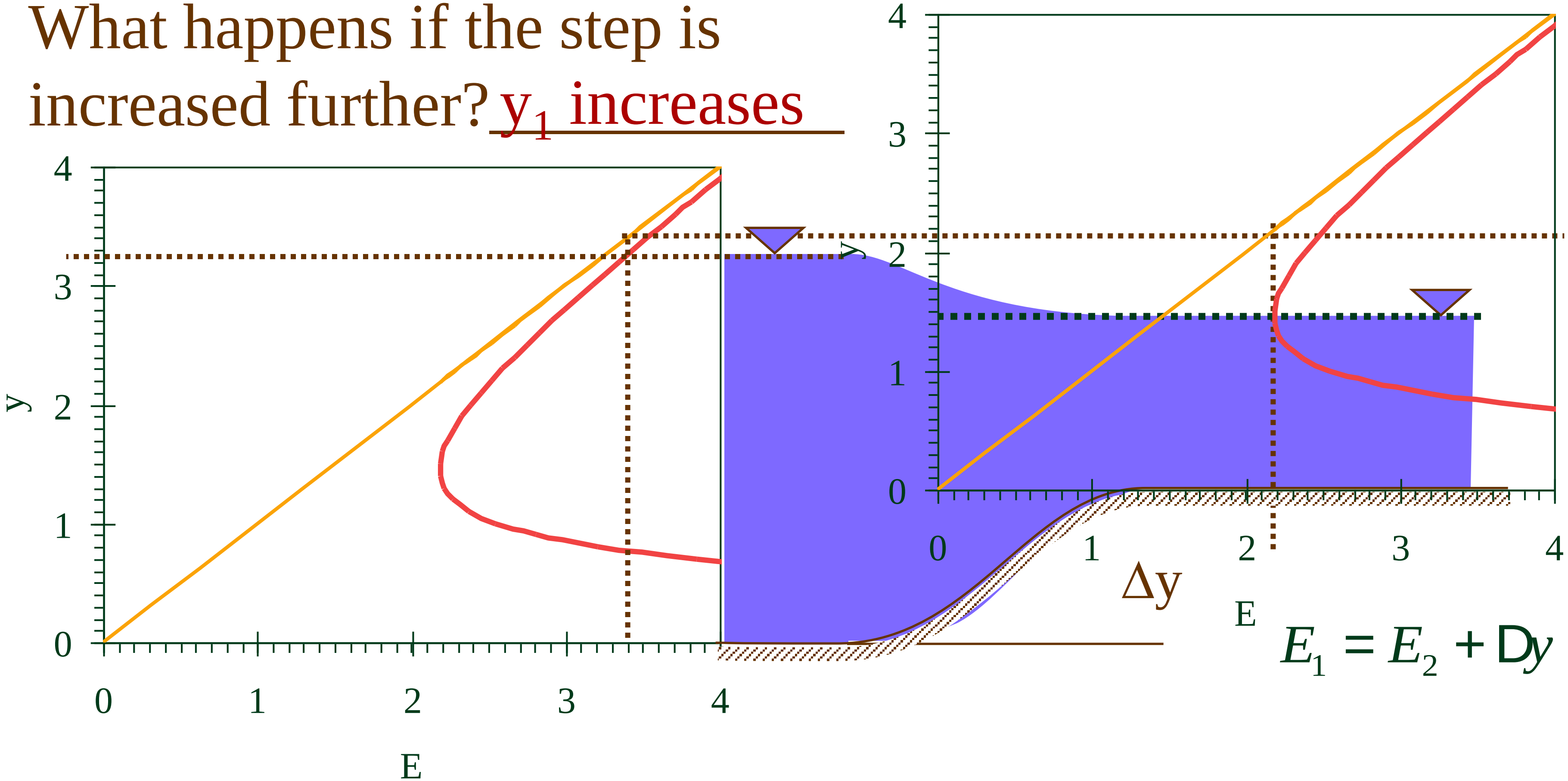


Is alternate depth possible? NO! Calculate depth along step.

Max Step Up

Short, smooth step with maximum rise Δy in channel

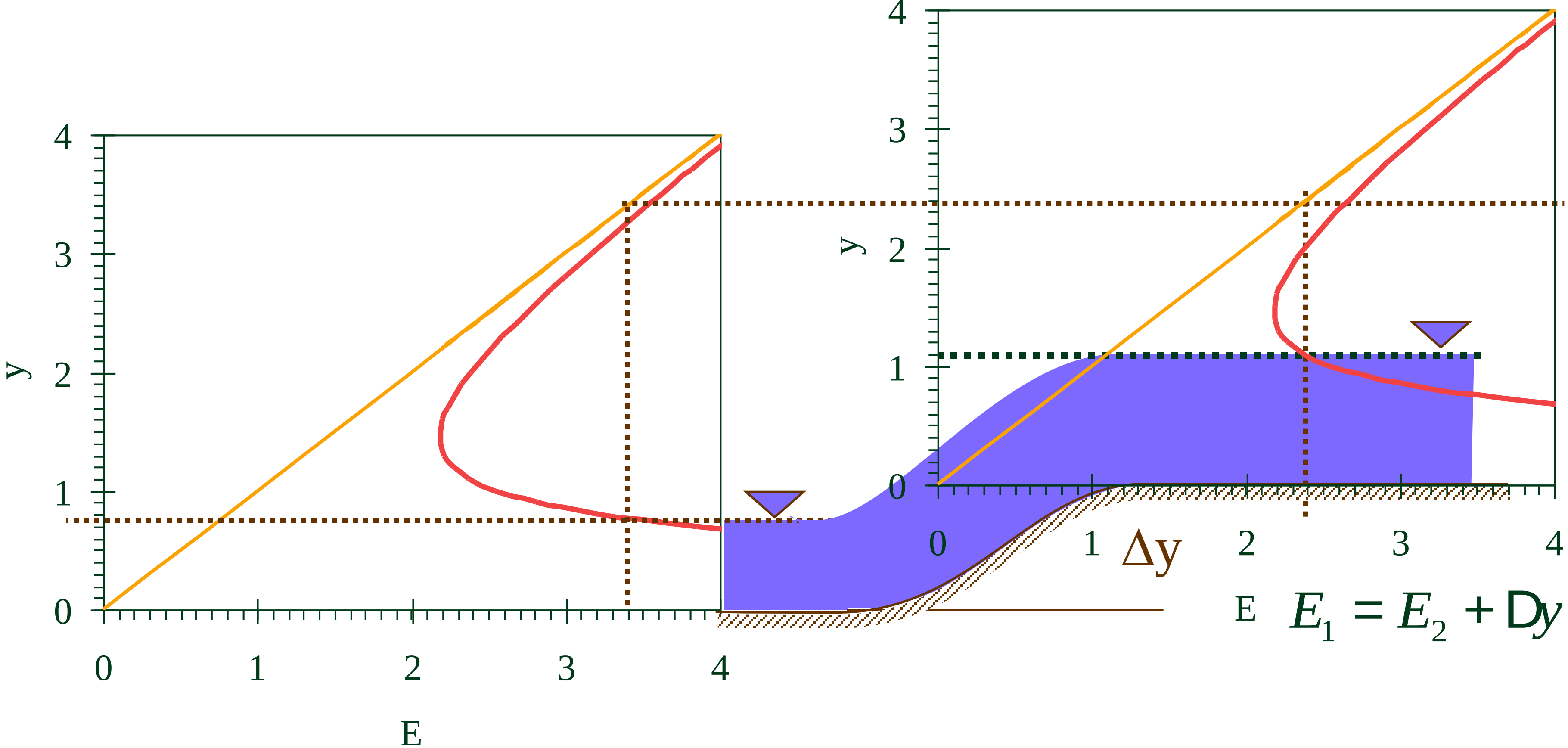
What happens if the step is increased further? y_1 increases



Step Up with Supercritical flow

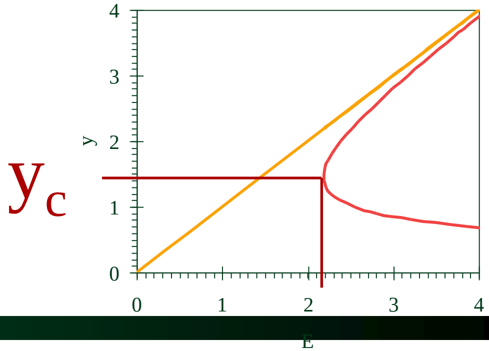
Short, smooth step with rise Δy in channel

Given upstream depth and discharge find y_2



What happened to the water depth? Increased! Expansion! Energy Loss

Critical Flow



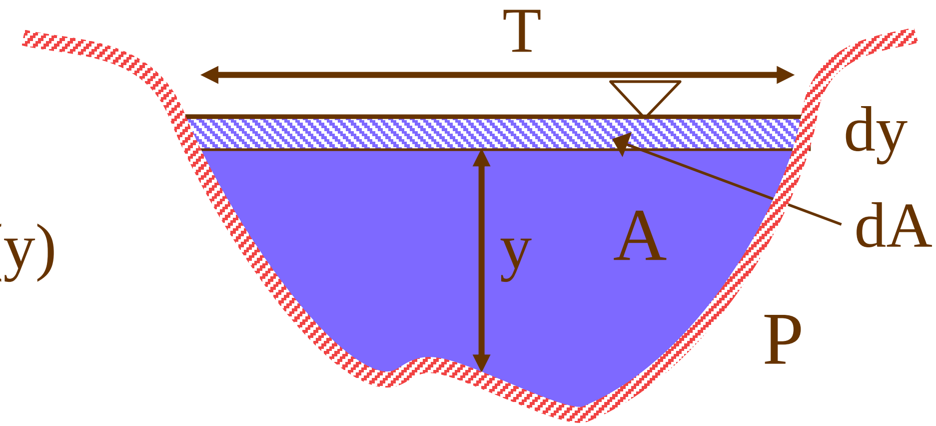
Find critical depth, y_c

$$\frac{dE}{dy} = 0$$

$$E = y + \frac{Q^2}{2gA^2}$$

Arbitrary cross-section

A=f(y)



$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} = 0$$

$dA = \underline{Tdy}$

T=surface width

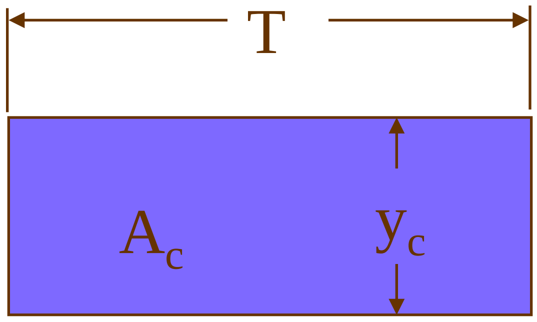
More general definition of Fr

$$1 = \frac{Q^2 T_c}{gA_c^3} \qquad \frac{Q^2 T}{gA^3} = Fr^2 \qquad \frac{V^2 T}{gA} = Fr^2 \qquad \frac{A}{T} = D$$

Hydraulic Depth

Critical Flow: Rectangular channel

$$1 = \frac{Q^2 T_c}{g A_c^3} \qquad T = T_c$$



$$Q = qT \qquad A_c = y_c T$$

$$1 = \frac{q^2 T^3}{g y_c^3 T^3} = \frac{q^2}{g y_c^3}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Only for rectangular channels!

$$q = \sqrt{g y_c^3}$$

Given the depth we can find the flow!

Critical Flow Relationships: Rectangular Channels

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c^3 = \left(\frac{V_c^2 y_c^2}{g}\right) \quad \text{because} \quad q = V_c y_c$$

$$\frac{V_c}{\sqrt{y_c g}} = 1$$

Froude number

$\dfrac{\text{inertial force}}{\text{gravity force}}$

$$\sqrt{\frac{Kinetic\ energy}{Potential\ energy}}$$

$$y_c = \frac{V_c^2}{g} \longrightarrow \frac{y_c}{2} = \frac{V_c^2}{2g}$$

velocity head = 0.5 (depth)

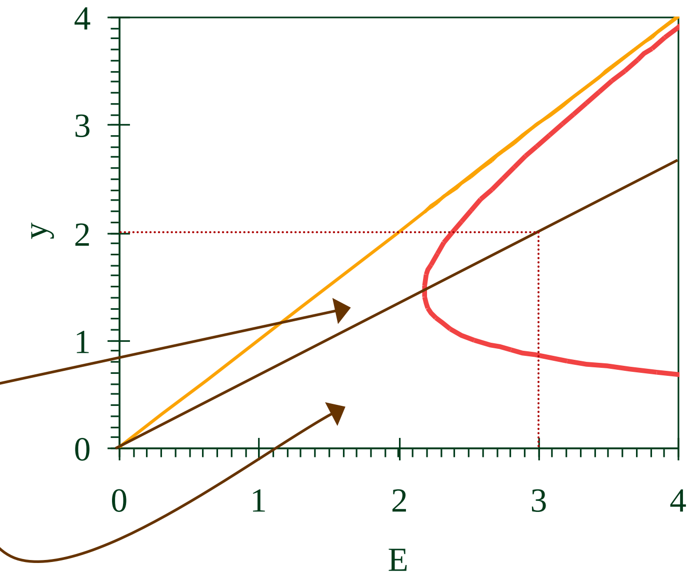
$$E = y + \frac{V^2}{2g} \longrightarrow E = y_c + \frac{y_c}{2} \longrightarrow y_c = \frac{2}{3}E$$

Critical Depth

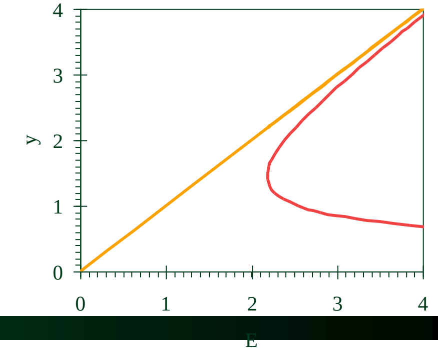
- Minimum energy for a given q
  - Occurs when $\frac{dE}{dy}$ = <u>0</u>
  - When kinetic = potential! $\frac{V_c^2}{2g} = \frac{y_c}{2}$
  - Fr=1

$$Fr = \frac{V_c}{\sqrt{y_c g}} = \frac{q}{\sqrt{g y_c^3}} = Q\sqrt{\frac{T}{gA^3}}$$

  - Fr>1 = <u>Super</u> critical
  - Fr<1 = <u>Sub</u> critical



Critical Flow



$$\frac{dE}{dy} = 0$$

- Characteristics
  - Unstable surface
  - Series of standing waves

Difficult to measure depth

- Occurrence
  - Broad crested weir (and other weirs)
  - Channel Controls (rapid changes in cross-section)
  - Over falls
  - Changes in channel slope from mild to steep
- Used for flow measurements
  - Unique relationship between depth and discharge

Broad-Crested Weir

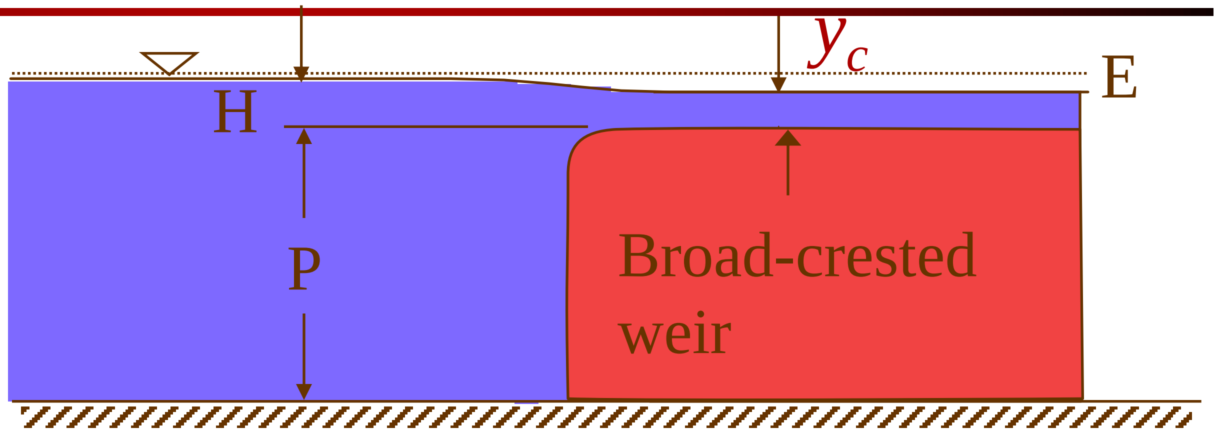
$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \sqrt{g y_c^3} \qquad Q = b\sqrt{g y_c^3}$$

$$y_c = \frac{2}{3}E$$

$$Q = b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} E^{3/2}$$

$$Q = C_d b\sqrt{g}\left(\frac{2}{3}H\right)^{3/2}$$



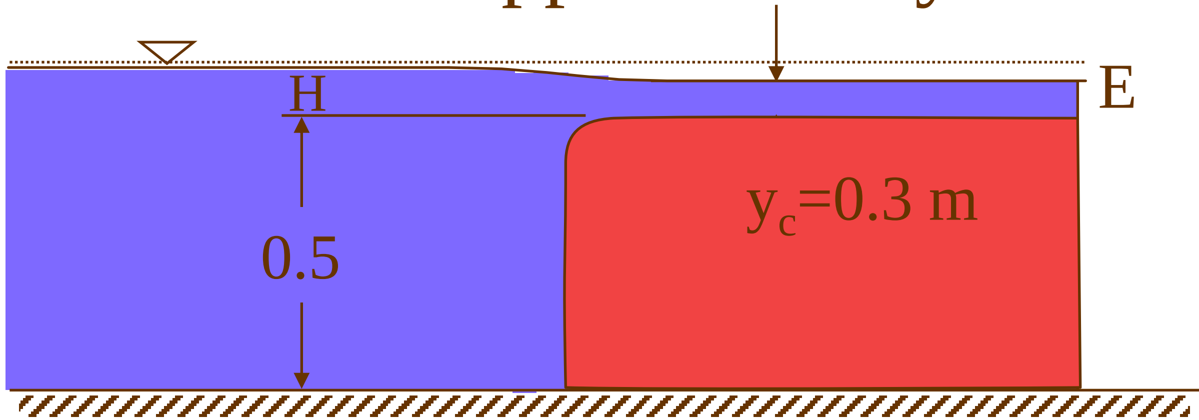
Hard to measure y_c

E measured from top of weir

C_d corrects for using H rather than E.

Broad-crested Weir: Example

- Calculate the flow and the depth upstream. The channel is 3 m wide. Is H approximately equal to E?



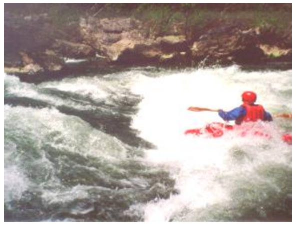
How do you find flow? Critical flow relation

How do you find H? Energy equation

Solution

Could a hydraulic jump be laminar?

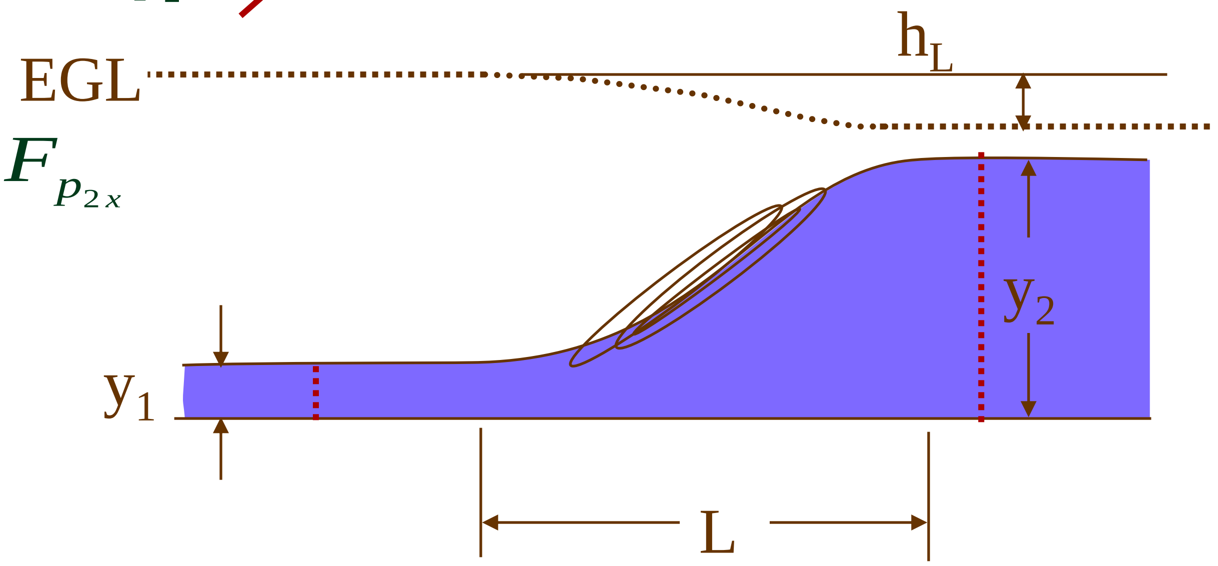
Hydraulic Jump



- Used for energy dissipation
- Occurs when flow transitions from supercritical to subcritical
  - base of spillway
  - Steep slope to mild slope
- We would like to know depth of water downstream from jump as well as the location of the jump
- Which equation, Energy or Momentum?

Hydraulic Jump

$$\mathbf{M}_1 + \mathbf{M}_2 = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$ Conservation of Momentum



$$M_{1x} + M_{2x} = F_{p_{1x}} + F_{p_{2x}}$$

$$M_{1x} = -\rho V_1^2 A_1$$

$$M_{2x} = \rho V_2^2 A_2$$

$$-\rho Q V_1 + \rho Q V_2 = \bar{p}_1 A_1 - \bar{p}_2 A_2$$

$$-\frac{Q^2}{A_1} + \frac{Q^2}{A_2} = \frac{g y_1 A_1}{2} - \frac{g y_2 A_2}{2}$$

$$\bar{p} = \frac{r\, g y}{2}$$

$$V = \frac{Q}{A}$$

Hydraulic Jump: Conjugate Depths

For a rectangular channel make the following substitutions

$$A = By \qquad Q = By_1V_1$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}}$$ Froude number

Much algebra ⟶ $$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8Fr_1^{\,2}}\right)$$

$$\frac{y_2}{y_1} = \frac{-1 + \sqrt{1 + 8Fr_1^{\,2}}}{2}$$

valid for slopes < 0.02

Hydraulic Jump: Energy Loss and Length

- Energy Loss $E_1 = E_2 + h_L$

$$E = y + \frac{q^2}{2gy^2} \xrightarrow{\text{algebra}} h_L = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

significant energy loss (to turbulence) in jump

- Length of jump

No general theoretical solution

Experiments show

$$L = 6y_2 \quad \text{for} \quad 4.5 < Fr_1 < 13$$

Specific Momentum

$$\frac{gy_1A_1}{2}+\frac{Q^2}{A_1}=\frac{gy_2A_2}{2}+\frac{Q^2}{A_2}$$

$$\frac{y_1A_1}{2}+\frac{Q^2}{A_1g}=\frac{y_2A_2}{2}+\frac{Q^2}{A_2g}$$

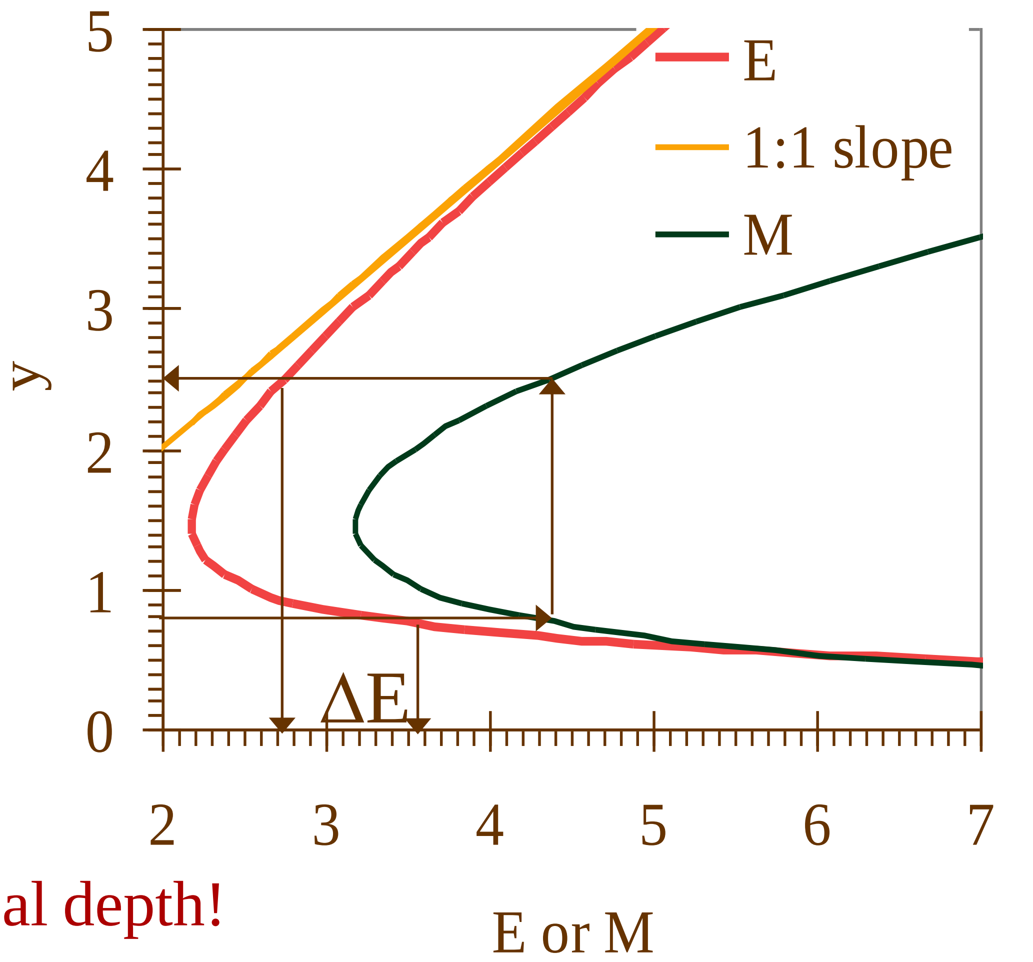
$$\frac{y_1^2}{2}+\frac{q^2}{y_1g}=\frac{y_2^2}{2}+\frac{q^2}{y_2g}$$

When is M minimum?

$$\frac{dM}{dy}=y+\frac{-q^2}{y^2g}$$

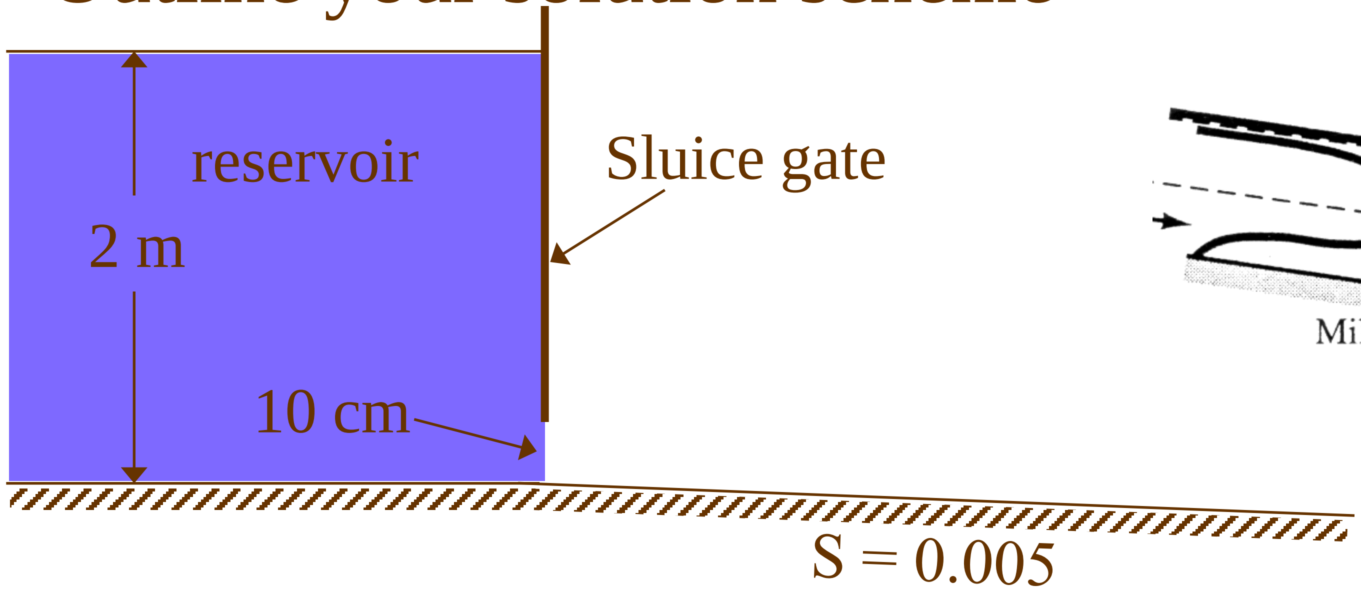
$$y=\left(\frac{q^2}{g}\right)^{1/3}$$

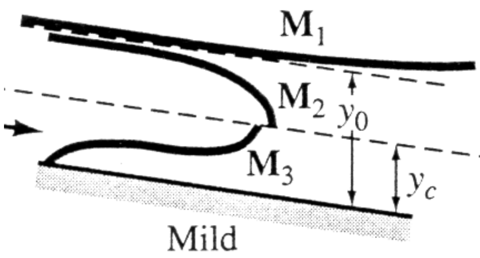
Critical depth!



Hydraulic Jump Location

- Suppose a sluice gate is located in a long channel with a mild slope. Where will the hydraulic jump be located?
- Outline your solution scheme





Gradually Varied Flow: Find Change in Depth wrt x

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Energy equation for non-uniform, steady flow

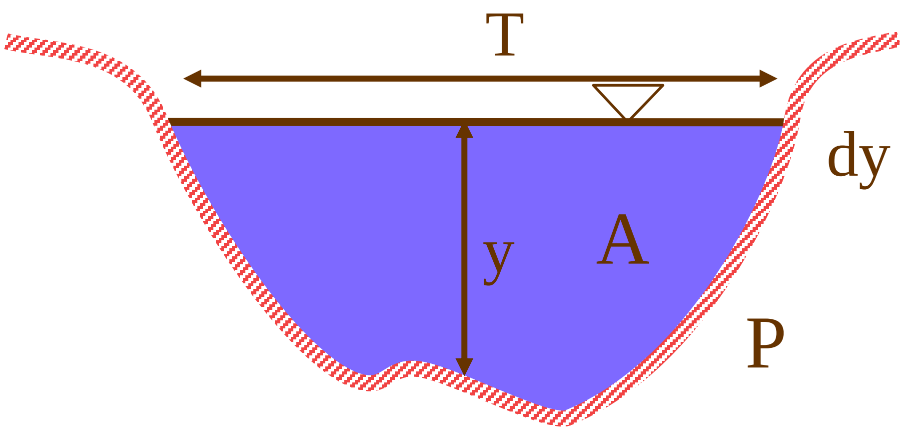
$$S_o dx = (y_2 - y_1) + \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) + S_f dx$$

$$dy = y_2 - y_1$$

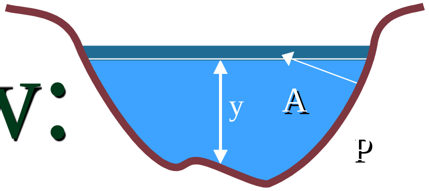
$$dy + d\left(\frac{V^2}{2g}\right) + S_f dx = S_o dx$$

$$\frac{dy}{dy} + \frac{d}{dy}\left(\frac{V^2}{2g}\right) + S_f \frac{dx}{dy} = S_o \frac{dx}{dy}$$

Shrink control volume



Gradually Varied Flow: Derivative of KE wrt Depth



$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = \left(\frac{-2Q^2}{2gA^3}\right)\cdot\frac{dA}{dy} = \left(\frac{-Q^2T}{gA^3}\right) = -Fr^2$$

$$\frac{dy}{dy} + \frac{d}{dy}\left(\frac{V^2}{2g}\right) + S_f\frac{dx}{dy} = S_o\frac{dx}{dy}$$

Change in KE / Change in PE

$$dA = Tdy$$

We are holding Q constant!

$$1 - Fr^2 + S_f\frac{dx}{dy} = S_o\frac{dx}{dy}$$

Does V=Q/A? Is V ⊥ A?

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

The water surface slope is a function of: bottom slope, friction slope, Froude number

Gradually Varied Flow: Governing equation

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

Governing equation for gradually varied flow

- Gives change of water depth with distance along channel
- Note
  - S_o and S_f are positive when sloping down in direction of flow
  - y is measured from channel bottom
  - dy/dx =0 means water depth is constant

y_n is when $S_o = S_f$

Surface Profiles

- **M**ild slope ($y_n > y_c$)
  - in a long channel subcritical flow will occur
- **S**teep slope ($y_n < y_c$)
  - in a long channel supercritical flow will occur
- **C**ritical slope ($y_n = y_c$)
  - in a long channel unstable flow will occur
- **H**orizontal slope ($S_o = 0$)
  - y_n undefined
- **A**dverse slope ($S_o < 0$)
  - y_n undefined

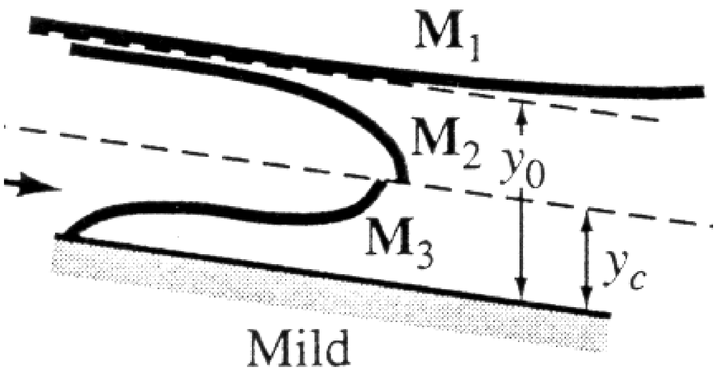
Note: These slopes are f(Q)!

Surface Profiles

Normal depth

Sluice gate

Steep slope

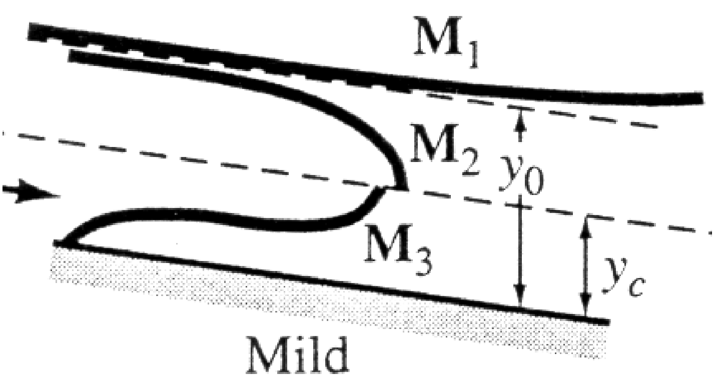


Obstruction

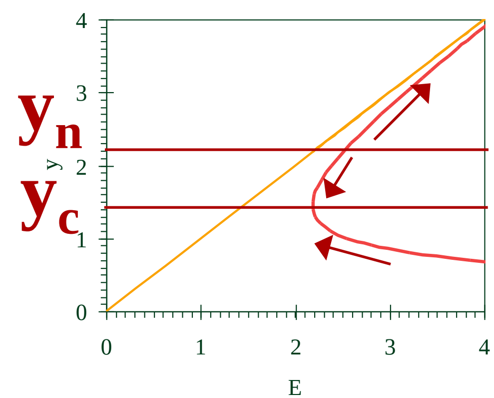
Steep slope (S_2)

Hydraulic Jump

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$



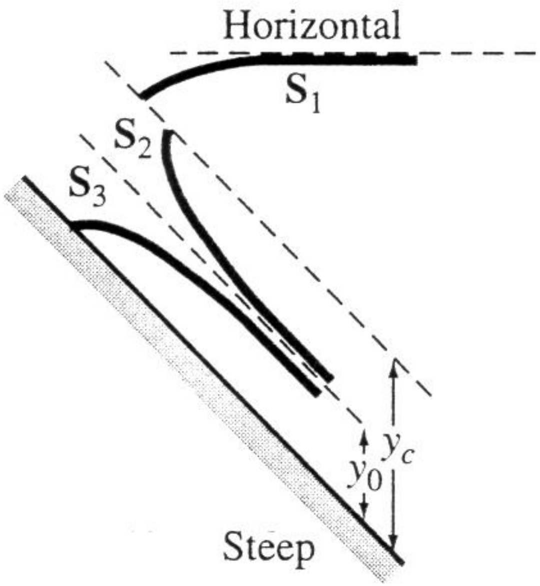
$S_0 - S_f$	$1 - Fr^2$	dy/dx
+	+	+
-	+	-
-	-	+

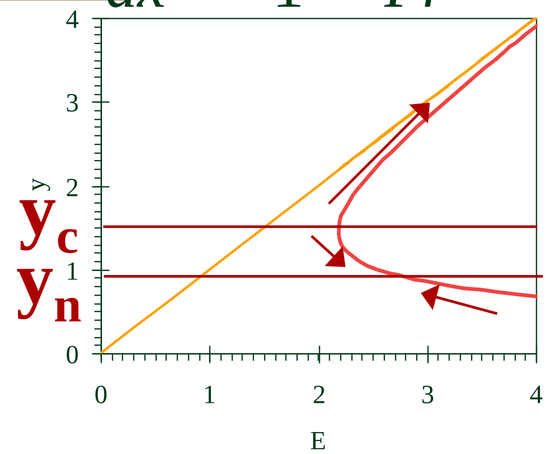


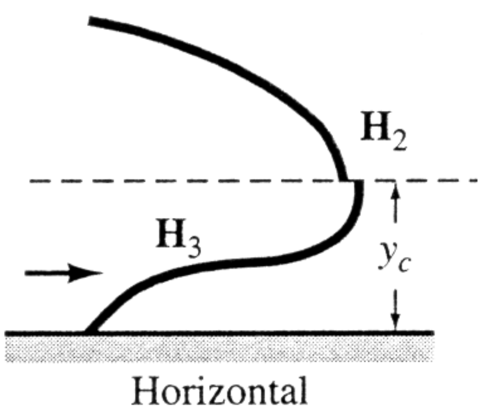
More Surface Profiles

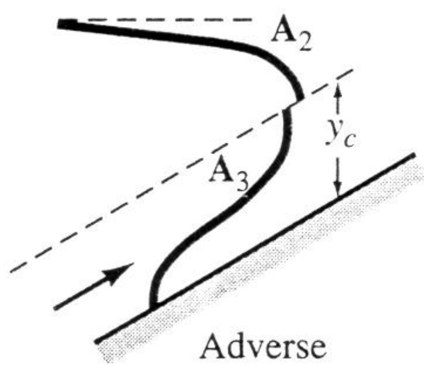
$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

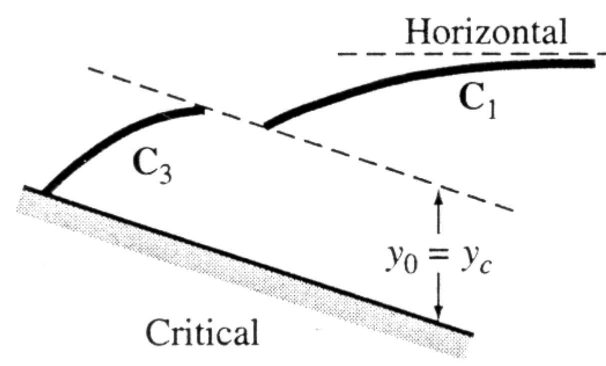
	$S_0 - S_f$	$1 - Fr^2$	dy/dx
1	+	+	+
2	+	-	-
3	-	-	+











Direct Step Method

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

energy equation

$$\Delta x = \frac{y_1 - y_2 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}}{S_f - S_o}$$

solve for Δx

rectangular channel

$$V_1 = \frac{q}{y_1} \qquad V_2 = \frac{q}{y_2}$$

prismatic channel

$$V_2 = \frac{Q}{A_2} \qquad V_1 = \frac{Q}{A_1}$$

Direct Step Method
Friction Slope

Manning		Darcy-Weisbach
$$S_f = \frac{n^2V^2}{R_h^{4/3}}$$	SI units	$$S_f = f\frac{V^2}{8gR_h}$$
$$S_f = \frac{n^2V^2}{2.22R_h^{4/3}}$$	English units	

Direct Step

- Limitation: channel must be prismatic (channel geometry is independent of x so that velocity is a function of depth only and not a function of x)
- Method
  - identify type of profile (determines whether Δy is + or -)
  - choose Δy and thus y_{i+1}
  - calculate hydraulic radius and velocity at y_i and y_{i+1}
  - calculate friction slope given y_i and y_{i+1}
  - calculate average friction slope
  - calculate Δx

Direct Step Method

=y*b+y^2*z

=2*y*(1+z^2)^0.5 +b

=A/P

=Q/A

=(n*V)^2/Rh^(4/3)

=y+(V^2)/(2*g)

=(G16-G15)/((F15+F16)/2-So)

$$\Delta x = \frac{y_1 - y_2 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}}{S_f - S_o}$$

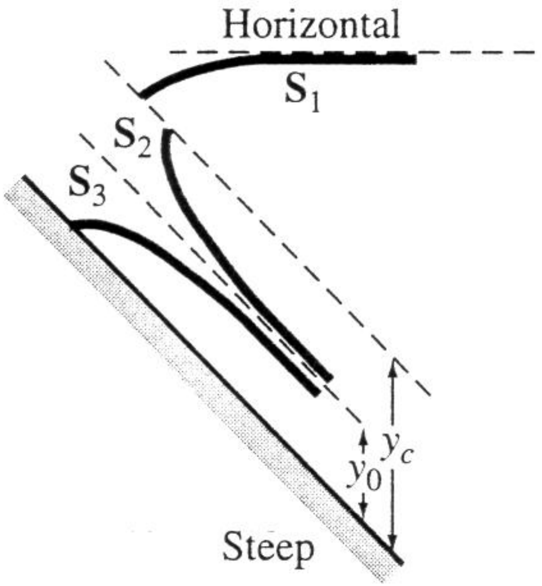
A	B	C	D	E	F	G	H	I	J	K	L	M
y	**A**	**P**	**Rh**	**V**	**Sf**	**E**	**Dx**	**x**	**T**	**Fr**	**bottom**	**surface**
0.900	1.799	4.223	0.426	0.139	0.00004	0.901		0	3.799	0.065	0.000	0.900
0.870	1.687	4.089	0.412	0.148	0.00005	0.871	0.498	0.5	3.679	0.070	0.030	0.900

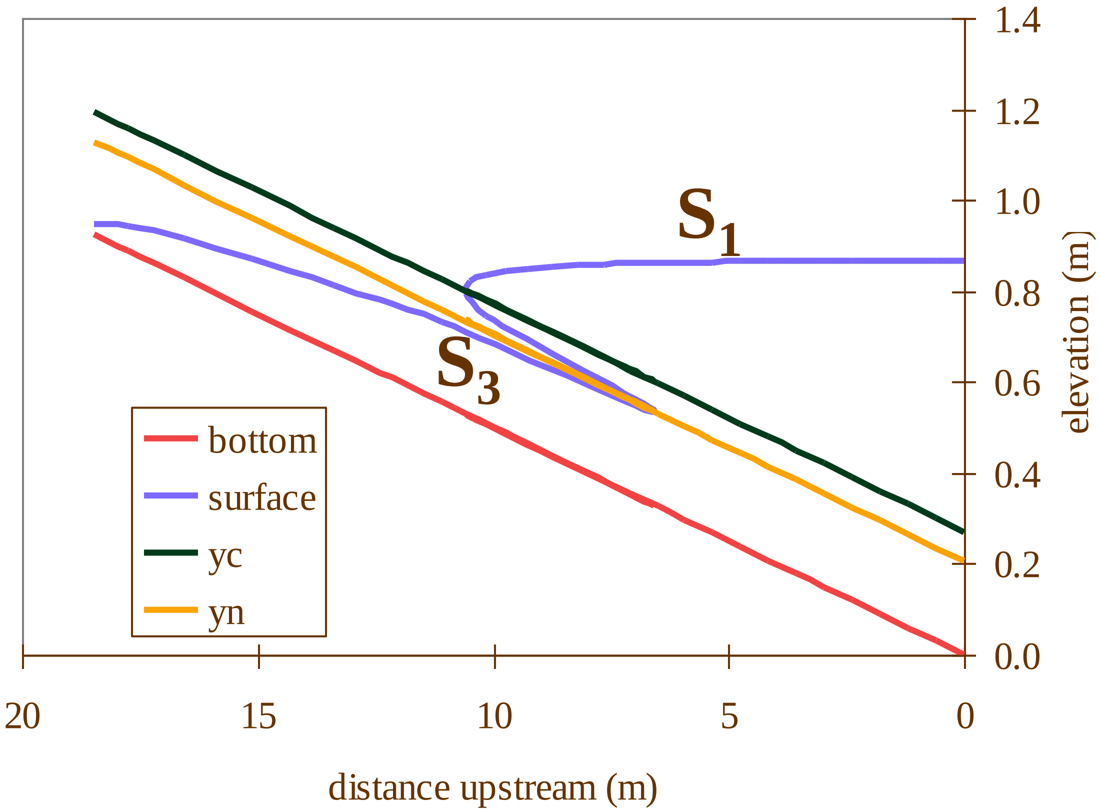
Standard Step

- Given a depth at one location, determine the depth at a second given location
- Step size (Δx) must be small enough so that changes in water depth aren't very large. Otherwise estimates of the friction slope and the velocity head are inaccurate
- Can solve in upstream or downstream direction
  - Usually solved upstream for subcritical
  - Usually solved downstream for supercritical
- Find a depth that satisfies the energy equation

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

What curves are available?
Steep Slope





Is there a curve between y_c and y_n that increases in depth in the downstream direction? __NO!__

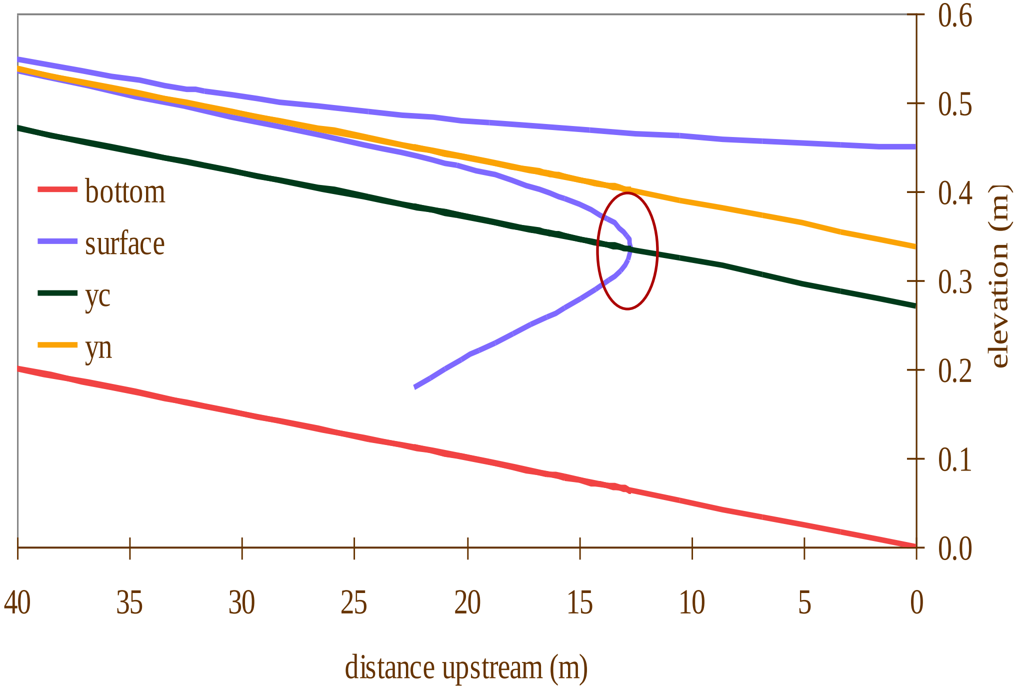
Mild Slope

- If the slope is mild, the depth is less than the critical depth, and a hydraulic jump occurs, what happens next?

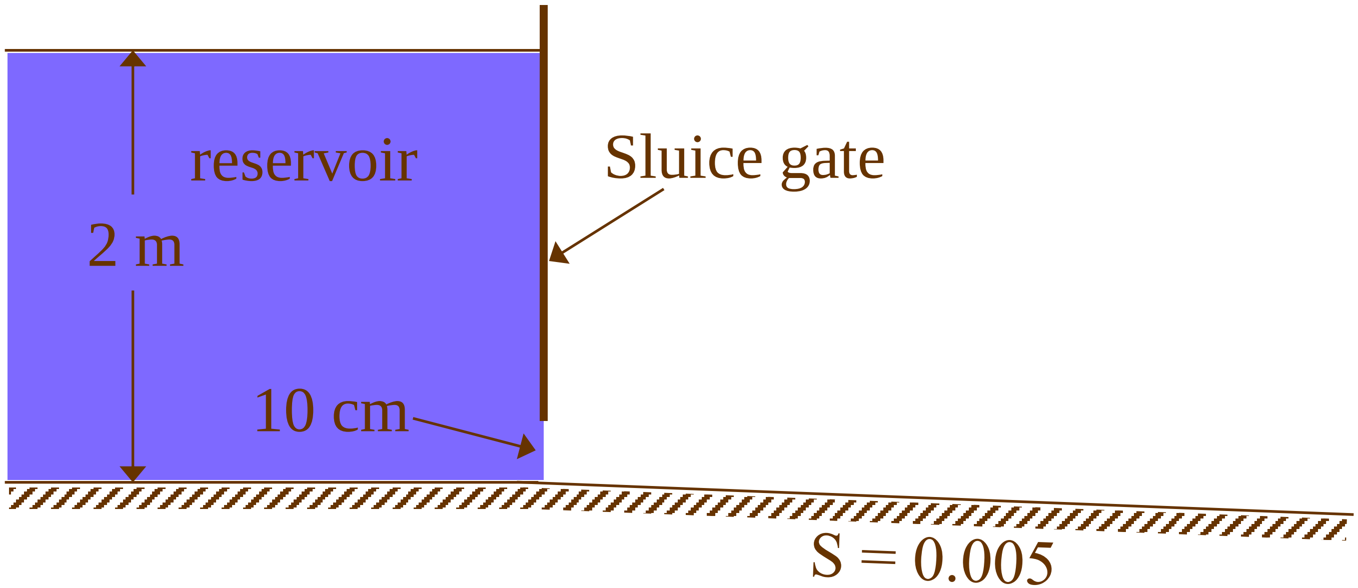
Rapidly varied flow!

When dy/dx is large then V isn't normal to cs

Hydraulic jump! Check conjugate depths

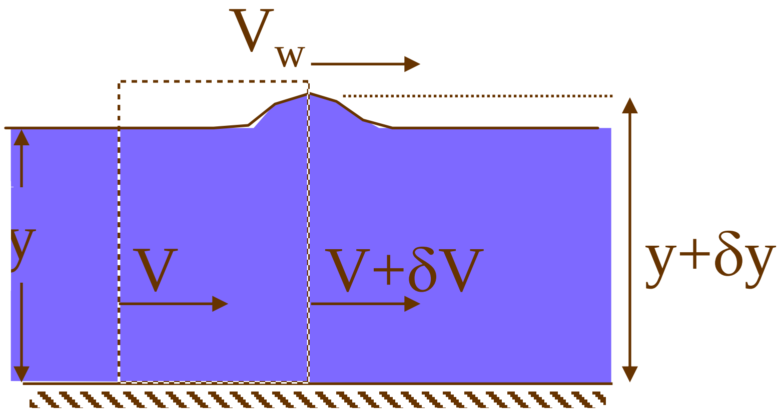


Water Surface Profiles: Putting It All Together

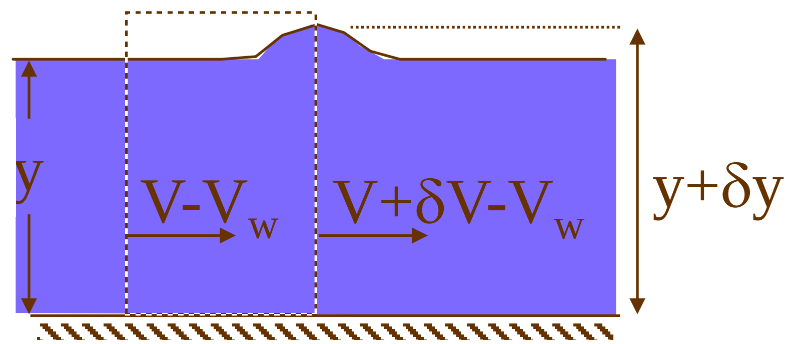


1 km downstream from gate there is a broad crested weir with P = 1 m. Draw the water surface profile.

Wave Celerity



unsteady flow



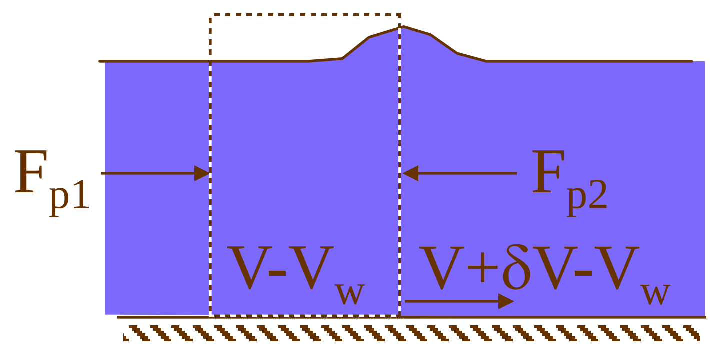
steady flow

$$\mathbf{M}_1 + \mathbf{M}_2 = \cancel{\mathbf{W}} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \cancel{\mathbf{F}_{ss}}$$

<u>Per unit width</u>

$$F_{p_1} = \frac{1}{2}\rho g y^2 \qquad F_{p_2} = -\frac{1}{2}\rho g (y+\delta y)^2$$

$$F_{p_1} + F_{p_2} = \frac{1}{2}\rho g \left[y^2 - (y+\delta y)^2 \right]$$



Wave Celerity: Momentum Conservation

$$M_1 = -\rho (V - V_w)^2 y \qquad M_2 = \rho (V + \delta V - V_w)(V - V_w) y$$

Per unit width

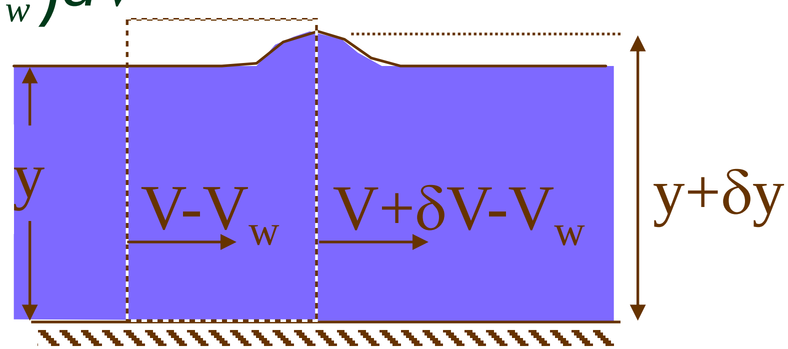
$$\mathbf{M}_1 + \mathbf{M}_2 = \rho y (V - V_w)\left[(\cancel{V} + \delta V - \cancel{V_w}) - (\cancel{V} - \cancel{V_w})\right]$$

$$\mathbf{M}_1 + \mathbf{M}_2 = \rho y (V - V_w)\delta V \qquad F_{p_1} + F_{p_2} = \frac{1}{2}\rho g \left[y^2 - (y + \delta y)^2 \right]$$

Now equate pressure and momentum

$$\frac{1}{2}\cancel{\rho} g \left[\cancel{y^2} - \cancel{y^2} - 2\cancel{y}\delta y - \delta \cancel{y^2}\right] = \cancel{\rho}\cancel{y}(V - V_w)\delta V$$

$$-g\delta y = (V - V_w)\delta V$$



steady flow

Wave Celerity

$$y(V - V_w) = (y + \delta y)(V + \delta V - V_w)$$

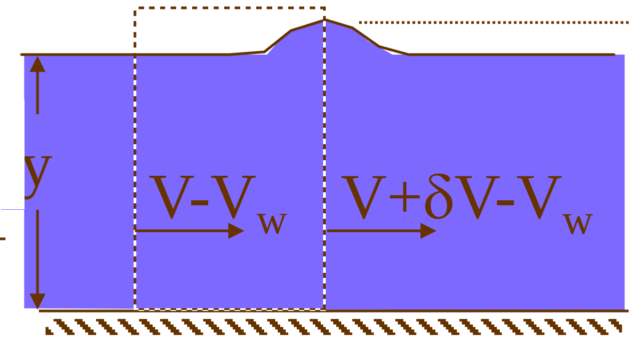
Mass conservation

$$yV - yV_w = yV + \delta yV + y\delta V + \delta y\delta V - yV_w - \delta yV_w$$

$$\delta V = -(V - V_w)\frac{\delta y}{y}$$

$$-g\delta y = (V - V_w)\delta V$$

Momentum



steady flow

$$g\delta y = (V - V_w)^2\frac{\delta y}{y}$$

$$gy = (V - V_w)^2 \qquad c = V - V_w \qquad c = \sqrt{gy}$$

$$\frac{V}{\sqrt{yg}} = Fr = \frac{V}{c}$$

Wave Propagation

- Supercritical flow
  - $c<V$
  - waves only propagate downstream
  - water doesn’t “know” what is happening downstream
  - _upstream_ control
- Critical flow
  - $c=V$
- Subcritical flow
  - $c>V$
  - waves propagate both upstream and downstream

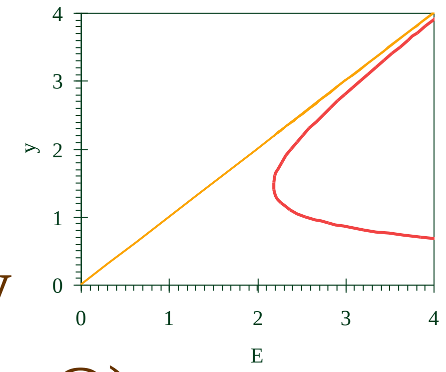
Discharge Measurements

- Sharp-Crested Weir $Q = \frac{2}{3} C_d b \sqrt{2g} H^{3/2}$
- V-Notch Weir $Q = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$
- Broad-Crested Weir $Q = C_d b \sqrt{g} \left(\frac{2}{3} H\right)^{3/2}$
- Sluice Gate $Q = C_d b y_g \sqrt{2g y_1}$

Explain the exponents of H! $V = \sqrt{2gH}$

Summary (1)

- All the complications of pipe flow plus additional parameter... free surface location
- Various descriptions of energy loss
  - Chezy, Manning, Darcy-Weisbach
- Importance of Froude Number
  - Fr>1 decrease in E gives increase in y
  - Fr<1 decrease in E gives decrease in y
  - Fr=1 standing waves (also min E given Q)



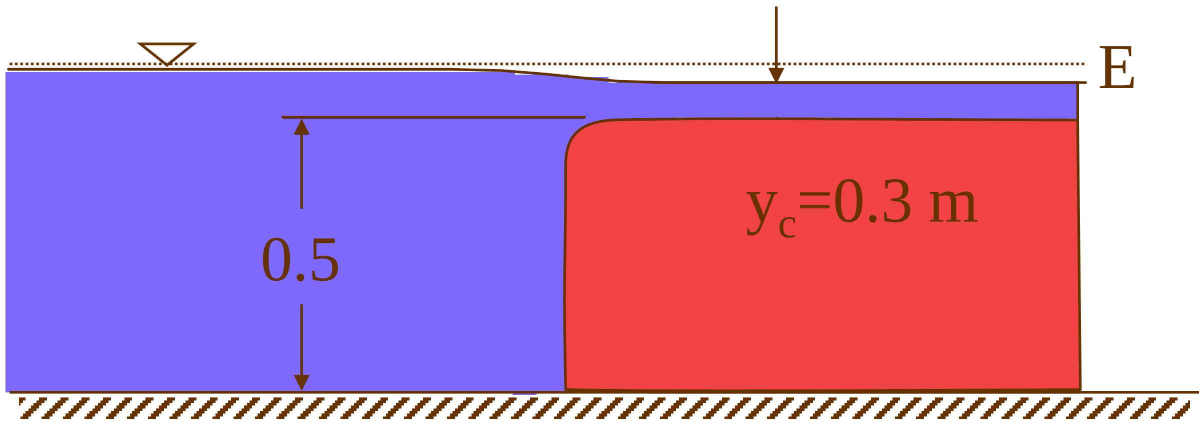
Summary (2)

- Methods of calculating location of free surface (Gradually varying)
  - Direct step (prismatic channel)
  - Standard step (iterative)
  - Differential equation

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

- Rapidly varying
  - Hydraulic jump

Broad-crested Weir: Solution



$$q = \sqrt{g y_c^3}$$

$$q = \sqrt{(9.8 m/s^2)(0.3 m)^3}$$

$$q = 0.5144 m^2/s$$

$$Q = qL = 1.54 m^3/s$$

$$y_c = \frac{2}{3} E$$

$$E_2 = \frac{3}{2} y_c = 0.45 m$$

$$E_1 = E_2 + P = 0.95 m$$

$$E_1 = y_1 + \frac{q^2}{2 g y_1^2}$$

$$E_1 - \frac{q^2}{2 g E_1^2} @ y_1 \longrightarrow y_1 = 0.935$$

$$H_1 = y_1 - 0.5 m = 0.435$$

Summary/Overview

- Energy losses
  - Dimensional Analysis
  - Empirical

$$V = \sqrt{\frac{8g}{f}}\sqrt{S_f R_h}$$

$$V = \frac{1}{n} R_h^{2/3} S_o^{1/2}$$

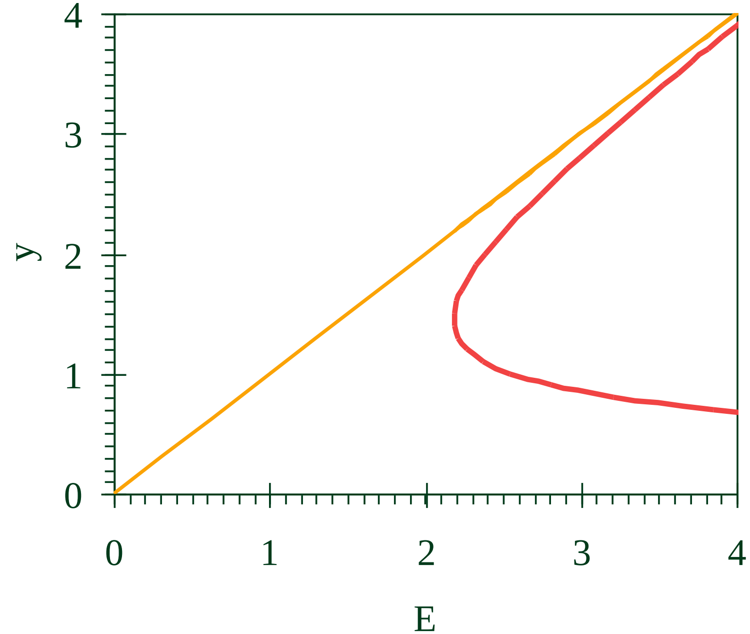
Energy Equation

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

- Specific Energy

$$E = y + \frac{V^2}{2g} = y + \frac{q^2}{2gy^2} = y + \frac{Q^2}{2gA^2}$$

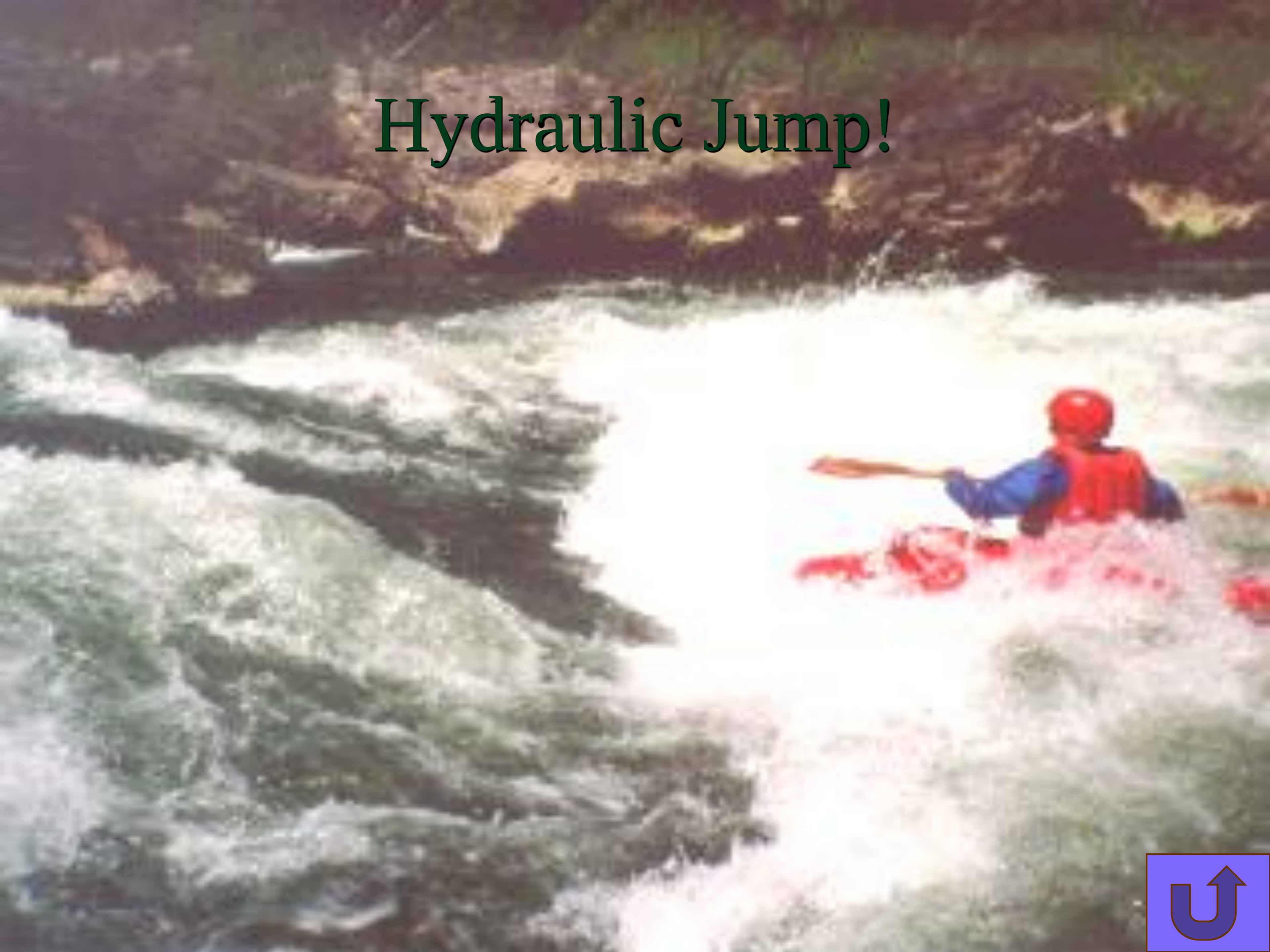
- Two depths with same energy!
  - How do we know which depth is the right one?
  - Is the path to the new depth possible?



What next?

- Water surface profiles
  - Rapidly varied flow
    - A way to move from supercritical to subcritical flow (Hydraulic Jump)
  - Gradually varied flow equations
    - Surface profiles
    - Direct step
    - Standard step

Hydraulic Jump!



Open Channel Reflections

- Why isn’t Froude number important for describing the relationship between channel slope, discharge, and depth for uniform flow?
- Under what conditions are the energy and hydraulic grade lines parallel in open channel flow?
- Give two examples of how the specific energy could increase in the direction of flow.